Lecture 3 – Exercises Solutions

Exercise 1: Conduction – properties and heat transfer through wall structures [L3, slides 9-10, 13-14]

1. Thermal conductivity of the materials considering the properties given can be found by combining Eqns. (3-11) and (3-13):

$$\boldsymbol{k} = \alpha \cdot C_p = \alpha \cdot \rho \cdot c_p$$

The calculated properties are listed in Table 1. $k_{concrete} > k_{wood} > k_{insulation}$ → the rate of heat transfer in concrete is ~3.5 times greater in wood and nearly 17 times higher than in insulation material. Thus, concrete is a better conductor among 3 materials, while expanded polystyrene is more efficient in reducing the heat flow (17 times better than concrete and 5 times better than wood). Wood is a better insulator than concrete.

2. Thermal admittance of the materials can be found using Eqn. (3-14a) or (3-14b).

$$\boldsymbol{\mu} = \sqrt{k \cdot C_p} \text{ or } \boldsymbol{\mu} = C_p\sqrt{\alpha}$$

The calculated properties are listed in Table 1. As $\boldsymbol{\mu}_{concrete} > \boldsymbol{\mu}_{fiberplaster} > \boldsymbol{\mu}_{wood} > \boldsymbol{\mu}_{insulation}$, concrete is the best material to store heat, while expanded polystyrene has minimum storage capacity. Therefore, the use of concrete as a wall core material would help to reduce the diurnal amplitude of indoor surface temperature fluctuations.

3. Total thermal transmittance of the wall assemblies can be found using Eqns. (3-6), (3-9)-(3-10):

$$U_{tot} = \frac{1}{R_{tot}}, R_{tot} = R_1 + R_2 + R_3 = \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}$$

For Wall 1, $R_{tot} = 6.02 \frac{\boldsymbol{m^2 \cdot K}}{\boldsymbol{W}}$ and $U_{tot} = 0.166 \frac{W}{m^2 \cdot K}$

For Wall 2, $R_{tot} = 6.03 \frac{\boldsymbol{m^2 \cdot K}}{\boldsymbol{W}}$ and $U_{tot} = 0.166 \frac{W}{m^2 \cdot K}$

As $U_{tot} < 0.17$ in both cases, the wall assemblies comply with the requirements of the norm SIA 380

Table 1: Composition of walls and thermal properties of layers

#	Layer	Material	L (m)	ρ $(\frac{kg}{m^3})$	c_p $(\frac{J}{kg \cdot K})$	$\alpha \times 10^{-6}$ $(\frac{m^2}{s})$	k $(\frac{W}{m \cdot K})$	μ $(\frac{J}{m^2 \cdot K \cdot s^{1/2}})$	R_i $(\frac{m^2 \cdot K}{W})$
Wall 1	1	fiber plaster	0.01	837	800	0.27	0.18	347.2	0.06
	2	concrete	0.15	1400	1000	0.36	0.5	836.7	0.30
	3	insulation	0.17	25	1380	0.87	0.03	32.2	5.67
Wall 2	1	fiber plaster	0.01	837	800	0.27	0.18	347.2	0.06
	2	wood	0.09	400	1255	0.28	0.14	265.1	0.64
	3	insulation	0.16	25	1380	0.87	0.03	32.2	5.33

4. As U_{tot} is identical for both wall structures, we can determine the heat flux only through one. Using Eqn. (3-8),

$$\dot{q} = U_{tot} \cdot \Delta T = 0.166 * (20 - 0) = 3.32\ W/m^2$$

5. Temperatures at the interfaces can be found using Fourier's law Eqn. (3-5b) as follows:
   - To find the temperature $\boldsymbol{t_2}$: $\dot{q} = \frac{1}{R_1} \cdot (T_1 - T_2) \rightarrow T_2 = T_1 - \dot{q} \cdot R_1$
   - To find the temperature $\boldsymbol{t_3}$: $\dot{q} = \frac{1}{R_2} \cdot (T_2 - T_3) \rightarrow T_3 = T_3 - \dot{q} \cdot R_2$

For both walls, $t_2 = 19.82$°C, while $t_3 = 18.82$°C for Wall 1 and $t_3 = 17.68$°C for Wall 2. Temperature change in a wooden wall is 2.37 K per 0.1 m, while only 0.66 K per 0.1 m in concrete.

NOTE: In parts (4) and (5), temperatures can be inserted in [**°C**] and not in [**K**] as the temperature differences are considered.

Exercise 2: Radiant heat flux from the window [L3, slide 22]

To determine radiant heat flux between the glass panes Eqns. (3-20)-(3-22) should be used:

$$\dot{q} = h_{rad,12} \cdot (T_2 - T_1) \text{ and } h_{rad,12} = 4 \cdot \varepsilon_{12} \cdot \sigma \cdot \left(\frac{T_1+T_2}{2}\right)^3 \text{ as } T_1 - T_2 < 50$$

a) For case A:

$$\frac{1}{\varepsilon_{12}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 = \frac{1}{0.88} + \frac{1}{0.88} - 1 = 1.27 \frac{W}{m^2 \cdot K} \rightarrow \boldsymbol{\varepsilon_{12} = 0.786}$$

$$\boldsymbol{h_{rad,12}} = 4 \cdot \varepsilon_{12} \cdot \sigma \cdot \left(\frac{T_1 + T_2}{2}\right)^3 = 4 \cdot 0.786 \cdot 5.67 \cdot 10^{-8} \cdot \left(\frac{292.15 + 265.15}{2}\right)^3 = \boldsymbol{3.86} \frac{W}{m^2 \cdot K}$$

$$\boldsymbol{\dot{q}} = h_{rad,12} \cdot (T_2 - T_1) = 3.86 \cdot (292.15 - 265.15) = \boldsymbol{104.22} \frac{W}{m^2}$$

b) For case A:

$$\frac{1}{\varepsilon_{12}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 = \frac{1}{0.88} + \frac{1}{0.09} - 1 = 11.25 \frac{W}{m^2 \cdot K} \rightarrow \boldsymbol{\varepsilon_{12} = 0.089}$$

$$\boldsymbol{h_{rad,12}} = 4 \cdot \varepsilon_{12} \cdot \sigma \cdot \left(\frac{T_1 + T_2}{2}\right)^3 = 4 \cdot 0.089 \cdot 5.67 \cdot 10^{-8} \cdot \left(\frac{292.15 + 265.15}{2}\right)^3 = \boldsymbol{0.44} \frac{W}{m^2 \cdot K}$$

$$\boldsymbol{\dot{q}} = h_{rad,12} \cdot (T_2 - T_1) = 0.44 \cdot (292.15 - 265.15) = \boldsymbol{11.88} \frac{W}{m^2}$$

c) Comparison:

When emissivity on the second pane is reduced from 0.88 to 0.09, effective emissivity reduces nearly 8.8 times, resulting in the same magnitude of the heat transfer coefficient and radiative heat flux reduction. Thus, applying the low-e coating on the second pane in case B is very successful in reducing radiant heat transfer.

Exercise 3: Surface radiation budget [L3, slide 31-32]

a) Net shortwave radiation budget per Eqns. (3-36) and (3-48):

$$\boldsymbol{K^*} = K_\downarrow - K_\uparrow = K_\downarrow - a \cdot K_\downarrow = 600 - 0.4 \cdot 600 = \mathbf{360\ W/m^2}$$

b) Net longwave radiation budget per Eqns. (3-37) and (3-39):

$$\boldsymbol{L^*} = L_\downarrow - L_\uparrow = 508.5 - 520 = \mathbf{-11.5\ W/m^2}$$

$$where\ \boldsymbol{L_\downarrow} = \frac{L_\uparrow - \varepsilon \cdot \sigma \cdot T_s^4}{1 - \varepsilon} = \frac{520 - 0.9 \cdot 5.67 \cdot 10^{-8} \cdot 309.65^4}{1 - 0.9} \approx \mathbf{508.5\ W/m^2}$$

c) Overall radiation budget per Eqn. (3-35)

$$\boldsymbol{Q^*} = K^* + L^* = 360 - 11.5 = \mathbf{348.5\ W/m^2}$$

The shortwave radiation budget $\boldsymbol{K^*}$ is much greater than the net longwave radiation budget $\boldsymbol{L^*}$ that has a negative contribution toward the overall radiation budget $\boldsymbol{Q^*}$ of the surface.